

Torsion, an alternative to dark matter?

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Abstract

We confront Einstein-Cartan's theory with the Hubble diagram. An affirmative answer to the question in the title is compatible with today's supernovae data.

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1 Introduction

The present day standard model of cosmology with a cosmological constant and pressureless dark matter fits observational data. However, despite quite some effort, no particle candidate for dark matter has been discovered. On the other hand, the standard model has only few challenging models with a finite number of parameters. We propose to use an Einstein-Cartan's theory as an alternative. The cosmological model we will discuss has the same number of parameters as the standard model. The dark matter density today of the standard model is replaced by a mean spin density of baryonic matter today.

Einstein-Cartan's theory [1] has several popular motivations:

- Space-time torsion, that Einstein puts to zero from the start, is allowed without however modifying the Einstein-Hilbert action. Consequently energy-momentum is still the source of space-time curvature with Newton's constant being the coupling constant and the source of torsion is half-integer spin with the same coupling constant.
- To describe matter with half-integer spin, one must use an orthonormal frame and the spin connection ω . Consequently the gauge invariance of the Einstein-Hilbert action is manifest with the gauge group being the Lorentz group. Furthermore, Einstein-Cartan's theory treats this connection as an independent field variable besides the metric. Both features look like promising steps towards unification of gravity with the standard model of particle physics, which is a gauge theory with a connection as independent field variable.
- Torsion closes the diagram of Figure 1. Let us explain the two arrows labeled 'geometry'. One possibility to define and measure curvature is by parallel transporting a vector around an infinitesimal geodesic parallelogram. The Riemann tensor evaluated on the two vectors defining the parallelogram is precisely the infinitesimal rotation mapping the transported vector after the round trip onto the initial vector. Torsion can be defined and measured in a similar fashion. In presence of non-vanishing torsion, the infinitesimal geodesic parallelogram does not close. The translation from the final to the initial point of the 'parallelogram' defines a tangent vector, which is precisely the torsion tensor evaluated on the two vectors defining the 'parallelogram'.

Since a long time physicists have been attracted by the beauty of torsion and there is a vast body of literature on the subject including applications in cosmology. Let us cite *the* historical review by Hehl, von der Heyde, Kerlick & Nester [2] (which was the second author's first contact with torsion), two recent reviews [3] and a few contributions of dynamical torsion in cosmology [4, 5]. Other results directly related to the present work with its non-propagating torsion will be referred to as we go along.

Let us mention that supergravity, which has been remarkably popular, is an Einstein-Cartan theory with matter being a spin $\frac{3}{2}$ Rarita-Schwinger field. If there torsion is put to zero, the theory loses its supersymmetry.

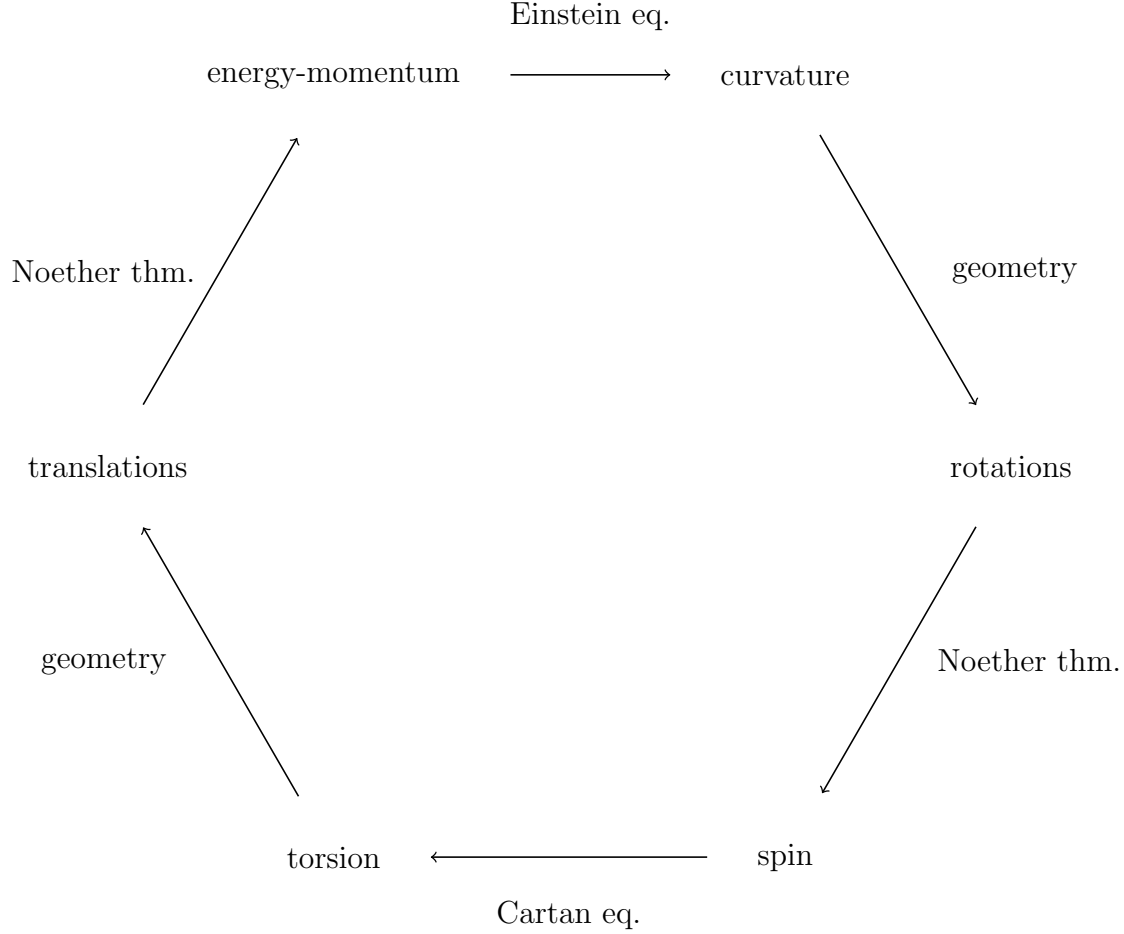


Figure 1: Einstein and Cartan's equations, geometric definition of curvature and torsion, Noether's theorem

Let us also mention two recent results concerning torsion in noncommutative geometry:

- (i) Torsion is produced together with curvature when the flat Dirac operator fluctuates [6] in the sense of Connes [7].
- (ii) Connes' spectral action [7] produces torsion terms together with the Einstein-Hilbert action [8].

2 Einstein-Cartan's theory in a nut shell

It is convenient to write Einstein-Cartan's theory using an orthonormal frame, *un repère mobile* using Cartan's words. Let x^μ ($\mu = 0, 1, 2, 3$) be a coordinate system on an open subset of \mathbb{R}^4 and let $e^a =: e^a_\mu dx^\mu$, $a = 0, 1, 2, 3$ be four 1-forms which are orthonormal with respect to a given space-time metric g . Following the conventions of reference [9], we write a metric connection with respect to this orthonormal frame as a 1-form with values in the Lie algebra of the Lorentz group, $\omega^a_b =: \omega^a_{b\mu} dx^\mu$. Then (suppressing all wedge

symbols) Cartan's two structure equations read:

$$R := d\omega + \frac{1}{2}[\omega, \omega], \quad (1)$$

for the curvature, a 2-form with values in Lie algebra of the Lorentz group, $R^a_b =: \frac{1}{2}R^a_{bcd}e^ce^d$, and

$$T := De = de + \omega e, \quad (2)$$

for the torsion, a vector-valued 2-form, $T^a =: \frac{1}{2}T^a_{bc}e^be^c$. It will be useful to decompose the torsion tensor into its three irreducible parts:

$$T_{abc} = A_{abc} + \eta_{ab}V_c - \eta_{ac}V_b + M_{abc}, \quad (3)$$

with the completely antisymmetric part $A_{abc} := \frac{1}{3}(T_{abc} + T_{cab} + T_{bca})$, the vector part $V_c := \frac{1}{3}T_{abc}\eta^{ab}$, and the mixed part M_{abc} characterized by $M_{abc} = -M_{acb}$, $M_{abc}\eta^{ab} = 0$, and $M_{abc} + M_{cab} + M_{bca} = 0$.

We have the two Bianchi identities:

$$DR = dR + [\omega, R] = 0, \quad (4)$$

$$DT = DDe = Re. \quad (5)$$

In these notations, the Einstein-Hilbert action reads

$$\begin{aligned} S_{\text{EH}}[e, \omega] &= \frac{1}{32\pi G} \int R^a_b * (e^be_a) - \frac{2\Lambda}{16\pi G} \int *1 \\ &= \frac{-1}{32\pi G} \int (R^{ab} + \frac{1}{6}\Lambda e^ae^b) e^ce^d \epsilon_{abcd} \\ &= \frac{-1}{16\pi G} \int (R^{ab}_{ab} + 2\Lambda) dV, \end{aligned} \quad (6)$$

where $*$ is the Hodge star of the metric g and $\epsilon_{0123} = 1$. The energy-momentum current is the vector-valued 3-form τ_a obtained by varying the orthonormal frame in the matter Lagrangian:

$$\mathcal{L}_M[e + f, \omega] - \mathcal{L}_M[e, \omega] =: -f^a \tau_a + O(f^2). \quad (7)$$

The energy-momentum tensor τ_{ab} is defined by $*\tau_a =: \tau_{ab}e^b$.

Likewise, the spin current is the Lorentz-valued 3-form S_{ab} obtained by varying the connection in the matter Lagrangian:

$$\mathcal{L}_M[e, \omega + \chi] - \mathcal{L}_M[e, \omega] =: -\frac{1}{2}\chi^{ab}S_{ab} + O(\chi^2). \quad (8)$$

The spin tensor S_{abc} is defined by $*S_{ab} =: S_{abc}e^c$.

Now we can derive the Einstein equation by varying the total action with respect to the orthonormal frame:

$$(R^{ab} + \frac{1}{3}\Lambda e^ae^b) e^d \epsilon_{abcd} = -16\pi G \tau_c \quad \text{or equivalently} \quad G_{ab} - \Lambda \eta_{ab} = 8\pi G \tau_{ab}, \quad (9)$$

with the Einstein tensor $G_{ab} := R^c_{acb} - \frac{1}{2}R^{cd}\eta_{ab}$.

Likewise Cartan's equation is derived by varying the total action with respect to the connection:

$$T^c e^d \epsilon_{abcd} = -8\pi G S_{ab}, \quad (10)$$

or equivalently:

$$A_{cab} + 3V_a \eta_{bc} - 3V_b \eta_{ac} + M_{cab} = -8\pi G S_{abc}. \quad (11)$$

Unlike curvature, torsion does not propagate: it is non-vanishing only inside matter with half-integer spin.

Note that in presence of general torsion, the Einstein tensor G_{ab} and energy-momentum tensor τ_{ab} are neither covariantly conserved nor symmetric. Indeed combining Einstein's equation with the first Bianchi identity, we have:

$$(R^{ab} + \Lambda e^a e^b) T^d \epsilon_{abcd} = -16\pi G (D\tau)_c, \quad (12)$$

and combining Cartan's equation with the second Bianchi identity, we have:

$$\tau_{ab} - \tau_{ba} = *(DS)_{ab}. \quad (13)$$

In absence of half-integer spin, general relativity is usually written with respect to a holonomic frame dx^μ and the metric g is given by its metric tensor $g_{\mu\nu}(x)$ with respect to the coordinates x^μ . The holonomic frame will be useful below to compute geodesics and invariant connections. The metric tensor reads $g_{\mu\nu}(x) = e^a_\mu(x) e^b_\nu(x) \eta_{ab}$. Traditionally the components of the connection with respect to a holonomic frame are denoted by $\Gamma^\alpha_\beta =: \Gamma^\alpha_{\beta\mu} dx^\mu$, a $gl(4)$ valued 1-form. The link between the components of the same connection with respect to the holonomic frame Γ and with respect to the orthonormal frame ω is given by the $GL(4)$ gauge transformation with $e(x) = e^a_\mu(x) \in GL(4)$;

$$\omega = e\Gamma e^{-1} + ede^{-1}, \quad (14)$$

or with indices:

$$\omega^a_{b\mu} = e^a_\alpha \Gamma^\alpha_{\beta\mu} e^{-1\beta}_b + e^a_\alpha \frac{\partial}{\partial x^\mu} e^{-1\alpha}_b. \quad (15)$$

The given connection (with or without torsion) is metric if 'angles' and 'lengths' are preserved under parallel transport. For the components ω of the connection this just means that its values ω^a_b are in the Lie algebra of the Lorentz group: $\omega_{ab} = -\omega_{ba}$. In terms of the holonomic components the metricity of the connection reads:

$$\frac{\partial}{\partial x^\lambda} g_{\mu\nu} - \Gamma^{\bar{\mu}}_{\mu\lambda} g_{\bar{\mu}\nu} - \Gamma^{\bar{\nu}}_{\nu\lambda} g_{\mu\bar{\nu}} = 0. \quad (16)$$

In presence of general torsion, there are two different geodesics $x^\mu(p)$. The first is defined by parallel transport of the velocity vector \dot{x} with the given connection:

$$\ddot{x}^\lambda + \Gamma^\lambda_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0, \quad (17)$$

where the over-dot denotes the derivative with respect to the affine parameter p . The second geodesic is defined by using the Christoffel connection of the given metric instead of the independent (metric) connection. This second geodesic minimizes locally the ‘arc length’. Both geodesics coincide if the torsion only has a completely antisymmetric part, $V = 0$, $M = 0$.

3 Vector fields preserving metric and connection

Let φ be a diffeomorphism with Jacobian matrix

$$\Lambda^{\bar{\mu}}_{\mu}(x) := \frac{\partial \varphi^{\bar{\mu}}}{\partial x^{\mu}}(x). \quad (18)$$

If $g_{\mu\nu}(x)$ is the metric tensor of a metric g with respect to the coordinates x^{μ} , then by definition φ is a (local) isometry if

$$g_{\mu\nu}(x) = (\Lambda^T)_{\mu}^{\bar{\mu}}(x) g_{\bar{\mu}\bar{\nu}}(\varphi(x)) \Lambda^{\bar{\nu}}_{\nu}(x). \quad (19)$$

Upon linearisation $\varphi(x) = x + \xi(x) + o(\xi^2)$, where $\xi = \xi^{\alpha} \partial/\partial x^{\alpha}$ is a vector field, equation (19) becomes the Killing equation:

$$\xi^{\alpha} \frac{\partial}{\partial x^{\alpha}} g_{\mu\nu} + \frac{\partial \xi^{\bar{\mu}}}{\partial x^{\mu}} g_{\bar{\mu}\nu} + \frac{\partial \xi^{\bar{\nu}}}{\partial x^{\nu}} g_{\mu\bar{\nu}} = 0. \quad (20)$$

Likewise if $\Gamma^{\lambda}_{\mu\nu}(x)$ are the components of a connection (not necessarily the Christoffel connection of the metric) with respect to the coordinates x^{μ} , then by definition φ preserves the connection if

$$\Gamma^{\lambda}_{\mu\nu}(x) = (\Lambda^{-1T})_{\bar{\lambda}}^{\lambda}(x) \Lambda^{\bar{\mu}}_{\mu}(x) \Lambda^{\bar{\nu}}_{\nu}(x) \Gamma^{\bar{\lambda}}_{\bar{\mu}\bar{\nu}}(\varphi(x)) - (\Lambda^T)_{\nu}^{\bar{\nu}}(x) \frac{\partial}{\partial x^{\mu}} (\Lambda^{-1T})_{\bar{\nu}}^{\lambda}(x). \quad (21)$$

Linearising we obtain:

$$\xi^{\alpha} \frac{\partial}{\partial x^{\alpha}} \Gamma^{\lambda}_{\mu\nu} - \frac{\partial \xi^{\lambda}}{\partial x^{\bar{\lambda}}} \Gamma^{\bar{\lambda}}_{\mu\nu} + \frac{\partial \xi^{\bar{\mu}}}{\partial x^{\mu}} \Gamma^{\lambda}_{\bar{\mu}\nu} + \frac{\partial \xi^{\bar{\nu}}}{\partial x^{\nu}} \Gamma^{\lambda}_{\mu\bar{\nu}} + \frac{\partial^2 \xi^{\lambda}}{\partial x^{\mu} \partial x^{\nu}} = 0. \quad (22)$$

4 The most general Riemann-Cartan space invariant under $O(3) \ltimes \mathbb{R}^3$

In order to combine Einstein-Cartan’s theory with the cosmological principle we have to solve the Killing equation (20), its analogue for the connection (22) and the metricity condition (16) for all vector fields generating the maximal isometry group of space. To simplify the calculations – and not because we particularly believe in flat spaces – we will take this group to be the 3-dimensional Euclidean group $O(3) \ltimes \mathbb{R}^3$ generated by three infinitesimal rotations $\xi = y\partial/\partial z - z\partial/\partial y$, $z\partial/\partial x - x\partial/\partial z$ and $x\partial/\partial y - y\partial/\partial x$, and by three infinitesimal translations $\xi = \partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$, where x, y, z are Cartesian coordinates.

The most general solution of the Killing equation is well known $d\tau^2 = h dt^2 - a^2(dx^2 + dy^2 + dz^2)$, with two positive functions $a(t)$ and $h(t)$. By a redefinition of the time coordinate t , we set $h \equiv 1$.

Equation (22) is the analogue of the Killing equation and describes connections invariant under a vector field ξ . Technically it consists of 64 differential equations for each vector field, of which some are empty, some are identical, some are algebraic. Our task therefore is straight-forward and tedious: write down and solve the 6×64 equations (22) coming from the six vector fields ξ . For the three translations we get immediately that all 64 components of Γ are independent of x , y and z . For the three rotations, it is sufficient to solve equation (22) for two infinitesimal rotations. It is then automatically satisfied for the third rotation because this rotation is the commutator of the two others. It remains to treat 2×64 equations. Finally, the most general connection solving equation (22) with the six vector fields has the following non-vanishing components:

$$\Gamma_{tt}^t = A(t), \quad \Gamma_{ij}^t = a(t)b(t)\delta_{ij}, \quad \Gamma_{jt}^i = c(t)\delta_{ij}, \quad (23)$$

$$\Gamma_{tj}^i = d(t)\delta_{ij}, \quad \Gamma_{jk}^i = f(t)\epsilon_{ijk}, \quad \epsilon_{123} = 1, \quad (24)$$

with five arbitrary functions of t : A, b, c, d, f . The metricity condition (16) yields with $' := d/dt$,

$$A = 0, \quad c = \frac{a'}{a}, \quad d = \frac{b}{a}. \quad (25)$$

So far we have only taken into account transformations close to the identity and we still have to discuss space inversion. Since a and b are parity-even and f is parity odd, we must set $f = 0$.

This connection reduces to the Christoffel connection if $b = a'$.

We use the gauge transformation (15) with $e^a_\mu = \text{diagonal}(1, a, a, a)$ to compute the non-vanishing holonomic components $\omega^a_{b\mu}$ of the connection:

$$\omega^0_{ij} = \omega^i_{0j} = b\delta_{ij}. \quad (26)$$

The Riemann tensor has the following non-vanishing components:

$$R^0_{i0k} = R^i_{00k} = \frac{b'}{a}\delta_{ik}, \quad R^i_{jkl} = \frac{b^2}{a^2}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}). \quad (27)$$

The Einstein tensor has:

$$G_{00} = 3\frac{b^2}{a^2}, \quad G_{ij} = -\left(2\frac{b'}{a} + \frac{b^2}{a^2}\right)\delta_{ij}. \quad (28)$$

Note that it is symmetric but not covariantly conserved.

The torsion tensor has:

$$T^i_{0j} = \frac{a' - b}{a}\delta_{ij}. \quad (29)$$

In particular the vector part has only a time component, $V_0 = (b - a')/a$, and the anti-symmetric and mixed parts vanish, $A = 0$, $M = 0$. This result agrees with the torsion found in references [10, 4] for spacetimes with maximally symmetric 3-spaces.

5 Hubble diagram

To make contact with physics, we must solve the geodesic equations for co-moving galaxies and for photons [11]. For both, torsion decouples and they reduce to the geodesic equations with the Christoffel connection of the metric. Consequently the redshift is still given by $z = a_0/a(t) - 1$ with look-back time t and the apparent luminosity ℓ is still related to the absolute luminosity of the standard candle L by

$$\ell(t) = \frac{L}{4\pi a_0^2 x(t)^2} \frac{a(t)^2}{a_0^2}. \quad (30)$$

We have put the earth at the origin of the Cartesian coordinates and the supernova on the x -axis:

$$x(t) := \int_t^{t_0} \frac{d\tilde{t}}{a(\tilde{t})}. \quad (31)$$

It is remarkable, that the Hubble diagram in absence of peculiar velocities is entirely determined by the metric and does not feel torsion. This is not true if the standard candle has non-vanishing peculiar velocity and/or if the messengers are massive neutrinos instead of photons.

On the other hand, the Einstein equations in presence of half-integer spin do feel torsion. Note however that the link between the Hubble constant H_0 and $d(z^2\ell)/dz(0)$ is purely kinematical. It does not depend on the dynamics, i.e. the Einstein equations. Therefore it does not depend on torsion. This fact will be crucial to identify consistently the initial conditions of the Einstein equations.

6 Matter, equations of state and field equations

Before we can write down the Einstein equations and see how they are modified by torsion, we must talk about their right-hand sides.

The most general energy-momentum tensor invariant under $O(3) \ltimes \mathbb{R}^3$ must satisfy an equation similar to the Killing equation (20). It therefore contains two arbitrary functions of cosmic time, $\rho(t)$ and $p(t)$ interpreted as energy density and pressure: $\tau_{\mu\nu} = \text{diag}(\rho, a^2 p, a^2 p, a^2 p)$. Note that we have already used the freedom of coordinate transformations to get rid of one function, $h(t)$, in the metric. We are therefore stuck now with two functions in the energy-momentum tensor. But then we have three functions, a, ρ, p and only two independent components in the Einstein tensor and must get rid of one function. This is usually done by an equation of state $p(t) =: w \rho(t)$. To simplify even more, one thinks of highly diluted particles and sets the pressure to zero, $w = 0$.

Likewise the most general $O(3) \ltimes \mathbb{R}^3$ -invariant spin tensor has one function of time $s(t)$ in the time component of the vector part, $s(t) := \frac{1}{3} S_{0ab} \eta^{ab}$. In Einstein-Cartan's cosmology for dust we then have four functions, a, b, ρ, s but only three equations, two of Einstein and one of Cartan. Therefore we need another equation of state. Our motivation is to do without dark matter. Visible matter consists mostly of protons, neutrons, electrons and

neutrinos, all of spin $\frac{1}{2}$ and all with small densities today. Therefore we keep neglecting the pressure. We also neglect spin-spin interactions and assume the spin density to be proportional to the particle density or energy density:

$$s(t) =: w_s \rho(t). \quad (32)$$

Note that, while w is dimensionless, w_s has units of time. For a single proton, we would expect $w_s \sim \hbar/(m_P c^2) \sim 10^{-25}$ s. For several protons with some spins anti-parallel, w_s could be even smaller. For a neutrino, w_s should be larger.

We have not tried to derive the spin density from any microscopic model. Kopczyński [12] assumed a Weyssenhoff fluid which has the draw-back of producing a spin-density incompatible with maximal space-symmetry. Böhmer & Burnett [13] studied Elko spinors or “dark” spinors as microscopic model. These spinors are *CPT*-odd and couple to ordinary matter only via gravity and via the Higgs mechanism, motivating the second terminology. Furthermore, they produce a spin-density compatible with the cosmological principle. Popławski [14] uses a particular form for the spin density, which he derives from quarks and leptons.

Now we are in position to write down the generalised Friedmann equations, i.e. the tt and the xx components of Einstein’s equations, and Cartan’s equation:

$$3 \frac{b^2}{a^2} = \Lambda + 8\pi G \rho, \quad (33)$$

$$2 \frac{b'}{a} + \frac{b^2}{a^2} = \Lambda, \quad (34)$$

$$3 \frac{a' - b}{a} = 8\pi G w_s \rho. \quad (35)$$

We have three equations for three unknown functions, a , b and ρ . The first equation (33) is algebraic, the other two equations, (34) and (35), are first order differential equations. We use the algebraic one to eliminate ρ from Cartan’s equation (35). Then we have a unique solution with two initial conditions $a(0) = a_0$ and $b(0) = b_0$. We therefore have four parameters, a_0 , b_0 , Λ and w_s . (We assume Newton’s constant known.) These four parameters are not independent. With equation (33) we now eliminate b_0 , Cartan’s equation then relates a_0 , Λ and w_s to the Hubble constant H_0 . The remaining equation (34) allows us to compute the deceleration parameter q_0 . We do not spell out this complicated expression for q_0 preferring to obtain the Hubble constant from the Hubble diagram at $z = 0$ (without using any dynamics) and to obtain the cosmological constant and the state parameter w_s from a fit of the entire Hubble diagram. Let us write the relation between the Hubble constant and a_0 , Λ and w_s :

$$1 = \Omega_{m0} + \Omega_{\Lambda 0} + 2\Omega_{s0} - \Omega_{s0}^2, \quad (36)$$

using familiar dimensionless quantities:

$$\Omega_m := \frac{8\pi G \rho}{3H^2} \quad \Omega_\Lambda := \frac{\Lambda}{3H^2} \quad \Omega_s := w_s H \frac{8\pi G \rho}{3H^2}. \quad (37)$$

In particular, we see that the scale factor today a_0 has dropped out. This is well-known for cosmology with vanishing spatial curvature and remains true in presence of non-vanishing torsion.

7 Data analysis

To confront Einstein-Cartan's theory with experimental data, we have used the so-called Union 2 sample [15] containing 577 type 1a Supernovae up to a redshift of 1.4. Our results have been obtained using the full covariance matrix including correlations and systematic errors. The magnitude evolution of supernovae with redshift is written as $M(z) = m_s - 2.5 \log \ell(z)$ where m_s is a normalisation parameter fitted to the data and $\ell(z)$ the apparent luminosity (30). As in standard cosmology, the magnitude evolution with redshift is Hubble-constant free.

The apparent luminosity is computed by solving the differential equations (34) and (35) where the algebraic equation (33) has been used to eliminate ρ . This system of 2 coupled differential equations is solved numerically by using the Runge-Kutta algorithm [16] with an adapted step in time such that the equivalent redshift step is much smaller than the experimental redshift error: $\Delta z < 10^{-5}$.

The best fitted cosmology is obtained using the MINUIT [17] package for the iterative χ^2 minimisation with:

$$\chi^2 = \Delta M^T V^{-1} \Delta M \quad (38)$$

where ΔM is the vector of differences between measured and expected magnitudes and V the full covariance matrix.

Since correlations between parameters are expected to be high, we marginalised over unwanted parameters and compute errors or contours by using the frequentist prescription [18] and solving $\chi^2 = \chi_{min}^2 + s^2$ ($s = 1$ for a 1σ error on a single variable) where χ_{min}^2 is the minimum χ^2 over all unwanted parameters. The Einstein-Cartan cosmology fit is performed with 3 free parameters (m_s, Ω_m, Ω_s) while Ω_Λ is a derived parameter obtained from the Friedmann-like equation (36). All of the following results are given with marginalisation over the nuisance parameter m_s .

Table 1 presents the results for the fit of Einstein-Cartan's theory and for comparison of the pure Einstein theory. To get the same number of free parameters, the fit for the pure Einstein theory does not use the flatness constraint. The minimum χ^2 for both fits are very close which implies that both theories are compatible with supernovae data at the same statistical level. The preferred value for Ω_s is 0.12 compatible with zero at a level better than 1 sigma. The effect of torsion is to lower the preferred value of Ω_m to 0.09 which is statistically compatible either with the baryon density of 0.046 ± 0.003 or the total matter density of 0.27 ± 0.03 published by the WMAP collaboration [19]. However, the spin density can contribute to the dark matter energy density to a certain amount. On the contrary the cosmological constant is only mildly modified by torsion.

Figure 2(b) shows probability contours in the (Ω_m, Ω_s) plane. As already mentioned, the degeneracy between both parameters is very high. As we can see, for negative spin

	Ω_m	Ω_Λ	Ω_s	χ^2_{min}
Einstein	$0.35^{+0.24}_{-0.29}$	$0.88^{+0.42}_{-0.55}$	0.	530.0
Einstein-Cartan	$0.09^{+0.30}_{-0.07}$	$0.83^{+0.10}_{-0.16}$	$0.12^{+0.02}_{-0.22}$	530.4

Table 1: Fit results (1σ errors) for Einstein and Einstein-Cartan theories. No flatness constraint is imposed in Einstein's theory.

density, the matter density increases while the cosmological constant density decreases (Figure 3). This suggests that a negative spin density may act as dark energy density.

We test this hypothesis by setting Ω_Λ equal to zero in the Einstein-Cartan equations and we redo the fit with only m_s and Ω_s as free parameters. In Figure 2(a) the green (lower) curve shows the result of the best fit for Einstein-Cartan theory fixing Ω_Λ to zero, while the red (upper) curve shows the best fit with free cosmological constant energy density. In the first case, the minimum χ^2 is equal to 1188 for a number of degrees of freedom of 575 while in the second case the minimum χ^2 is 530.4 for a number of degrees of freedom of 574. These results rule out the hypothesis of torsion to replace dark energy. Let us mention the analysis of reference [20], which contains a similar analysis using the parity-odd torsion [21]. They find that this part of torsion can mimic a cosmological constant.

In Figure 3 we show the probability contours in the $(\Omega_m, \Omega_\Lambda)$ plane for both Einstein-Cartan and Einstein theories. The degeneracy between both theories is almost orthogonal. The main reason is coming from the Friedmann-like equation for Einstein-Cartan theory. This constraint mimics an approximate flatness $\Omega_m + \Omega_\Lambda = 1$. We notice that both contours cross at a confidence level of 39% at the usual Λ CDM point, corresponding to vanishing torsion. From a purely statistical point of view, the actual supernovae data do not allow us to discriminate between both theories.

8 Conclusion

It is well known that curvature is not an alternative to dark matter in the Hubble diagram, see Figure 3. On the other hand, within the error bars of the supernova luminosities, torsion à la Einstein-Cartan allows us to fit the data with a matter contribution of $\Omega_m = 5\%$ corresponding to visible matter only. However the best fit has $\Omega_m = 9\%$ implying still a few percent of dark matter. More disturbingly, its corresponding state parameter is $w_s \sim H_0^{-1} \sim 10^{17}$ s, 42 orders of magnitude away from the naive value $w_s \sim \hbar/(m_P c^2) \sim 10^{-25}$ s.

To conclude, we think that the present result is encouraging enough to reconsider the rotation curves in galaxies and the CMB analysis in the light of the Einstein-Cartan theory.

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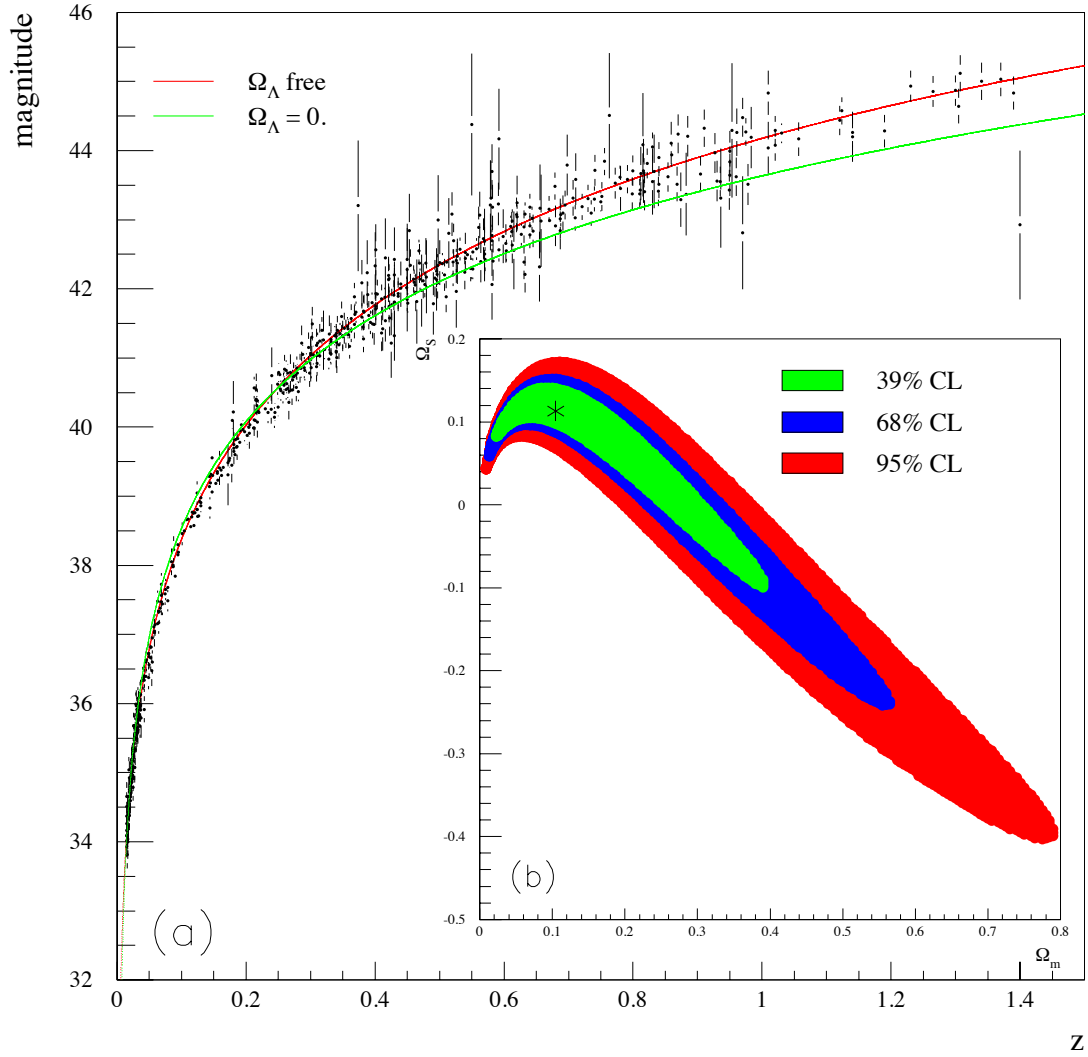


Figure 2: (a) Fit results using the Union 2 Supernovae sample. The red (upper) curve corresponds to the Einstein-Cartan 3-fit $(m_s, \Omega_m, \Omega_s)$ while the green (lower) curve represents the 2-fit assuming a vanishing cosmological constant. (b) 39%, 68% and 95% confidence level contour in the (Ω_m, Ω_s) plane for the Einstein-Cartan 3-fit.

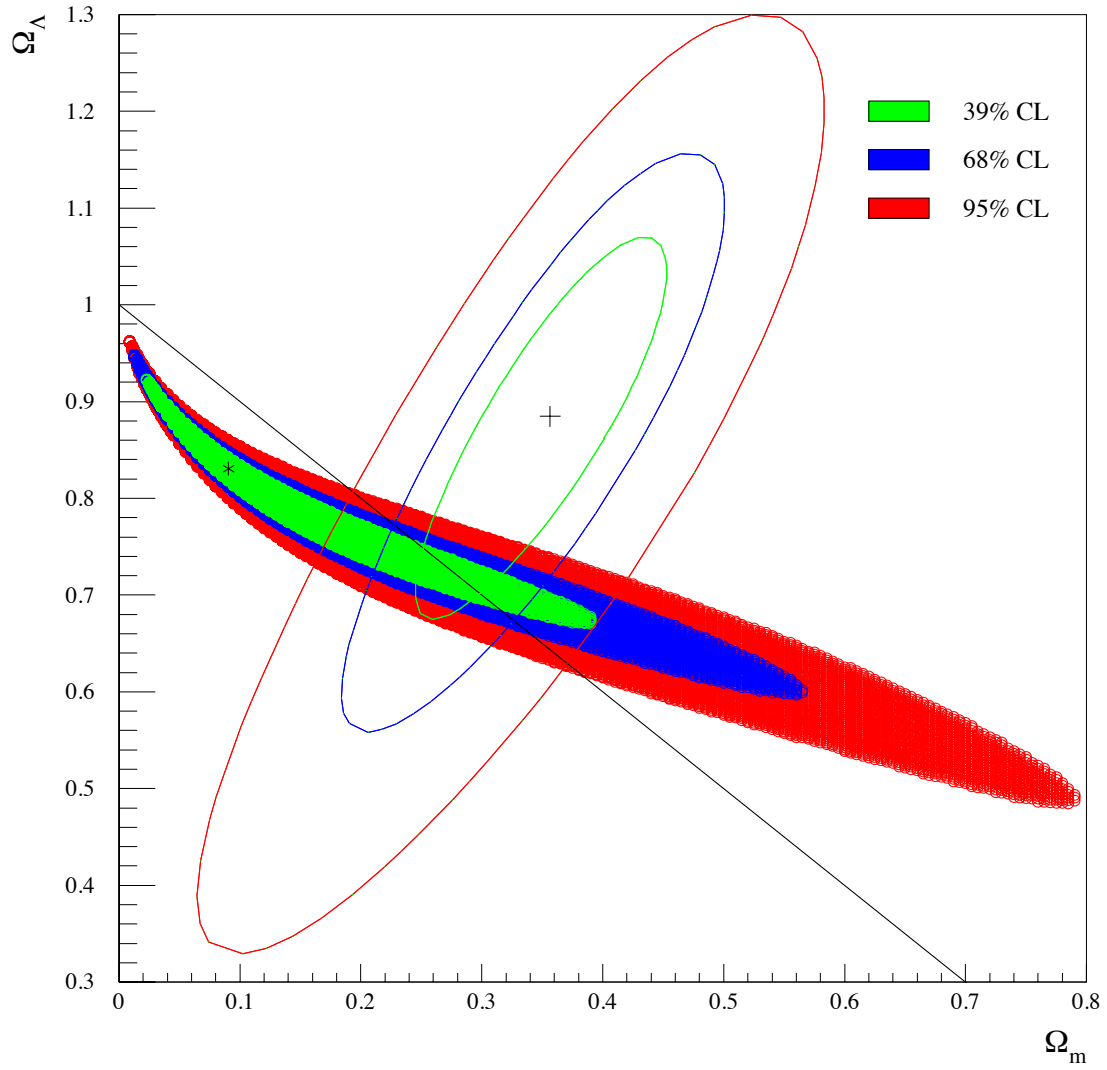


Figure 3: 39%, 68% and 95% confidence level contours in the $(\Omega_m, \Omega_\Lambda)$ plane for the Einstein-Cartan 3-fit (full contours) and for the Einstein 3-fit (full colored (elliptical) lines). The black (straight) line corresponds to the usual Einstein flatness.